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Parametric Surfaces in PHIGS PLUS: a New Chance for Patterning and Hatching?

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The Polygon Fill styles PATTERN and HATCH, which are quite successful in 2D graphics standards as GKS [1] and CGI [4], have proved to be less suitable for 3D graphics standards as GKS-3D [2] and PHIGS [3]. However, the emerging standard PHIGS PLUS [5] offers a unique chance to successfully employ these Interior Styles (under another name and in a slightly different form), because PHIGS PLUS supports inter alia topologically rectangular parametric surfaces. In this article it is shown how these Interior Styles could be efficiently applied to curved surfaces in PHIGS PLUS. In addition, the possible interaction between the tessellation method and the patterning is shown.
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1. Introduction

When GKS [1] was extended into GKS-3D [2] and later into PHIGS [3], one of the problems for the developers was how to support the Interior Style PATTERN in 3D graphics, without violating the upward compatibility with GKS.
Their efforts resulted in the new function SET PATTERN REFERENCE POINT AND PATTERN REFERENCE VECTORS plus the construction rules for the pattern box in § 4.4.7 of GKS-3D [2] or in § 4.5.8 of PHIGS [3]. However, although the result did satisfy the upward compatibility with GKS and made patterning possible, it did not make the use of Interior Style PATTERN attractive. It is, for example, very awkward to draw a surface, consisting of several adjacent and non-planar faces, with the Interior Style PATTERN. Every face will have a different pattern drawn on it and the resulting picture will probably be jarring to the eye.
For different reasons, drawing such a surface with the Interior Style HATCH, will lead to unsatisfactory results as well: all the hatch lines will have the same direction after the projection on the 2D plane, which will not contribute to a realistic picture.
In sections 2-5, we will show how the new functionality in PHIGS PLUS offers the possibility of drawing patterns and hatches on curved surfaces which are topologically rectangular.

2. Parametric Curves and Surfaces

PHIGS PLUS supports some new classes of output primitives. Two of these new types are the so-called Parametric Curves and Parametric Surfaces. Both primitives may also be defined in homogeneous coordinates.
The Parametric Curve is defined by

$$X(t) = C_1(t);$$

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$$Y(t) = C_2(t);$$

$$Z(t) = C_3(t);$$

$$[W(t) = C_4(t)];$$

$$C_i(t) = \sum_{k=0}^{n} C_{ik} N_k(t), \; i = 1, 2, 3 \; (, 4);$$

$$t_{min} \leqslant t \leqslant t_{max};$$

where $N_k(t)$ are piecewise polynomial shape functions in t , for example Non-Uniform Rational B-Splines (NURB) or Bézier splines.
The Parametric Surface is defined by

$$X(u,v) = S_1(u,v);$$

$$Y(u,v) = S_2(u,v);$$

$$Z(u,v) = S_3(u,v);$$

$$[W(u,v) = S_4(u,v)];$$

$$S_i(u,v) = \sum_{k=0}^{m} \sum_{l=0}^{n} S_{ikl} M_k(u) N_l(v);$$

$$i = 1, 2, 3 \; (, 4);$$

$$u_{min} \leqslant u \leqslant u_{max}; \; v_{min} \leqslant v \leqslant v_{max};$$

where $M_k(u)$ and $N_l(v)$ are piecewise polynomial shape functions in u and v , for example Non-Uniform Rational B-Splines (NURB) or Bézier splines. See [7] for an extensive description of NURB.

3. The PHIGS PLUS Interior Style GENERAL

With the currently available Interior Styles from PHIGS, curved surfaces in PHIGS PLUS can in practice only be drawn with the Interior Style SOLID. The use of the styles EMPTY or HOLLOW would produce a practically invisible surface and HATCH and PATTERN could only be applied to the tessellating faces, which would no doubt produce a rather exotic but unattractive figure.
In order to overcome this limitation, PHIGS PLUS [5] has introduced a completely new Interior Style: the style GENERAL. This style is a very powerful tool for drawing surfaces in other styles than SOLID.
Associated with this style is a *General Interior Representation,* which can be retrieved from the workstation's *General Interior Bundle Table* using the current Interior Style Index (see figure 1).
This General Interior Representation has the following entries:

1) the *General Interior Type,* an integer denoting how the surface is to be filled; currently, only the type ISOPARAMETRIC CURVES is supported by PHIGS PLUS; however, more Interior Types may added in future versions of PHIGS PLUS;
2) the data record associated with the General Interior Type.

4. The General Interior Type ISOPARAMETRIC CURVES

The purpose of this General Interior Type is to support the wire-frame model for curved 3D surfaces: with the current functionality of PHIGS, it is possible but not simple to draw curved surfaces as wire-frames.
This General Interior Type supports the wire-frame model in the following way (see also figure 2):
Let

$$\vec{S}(u,v), \; u_{min} \leqslant u \leqslant u_{max}, \; v_{min} \leqslant v \leqslant v_{max};$$

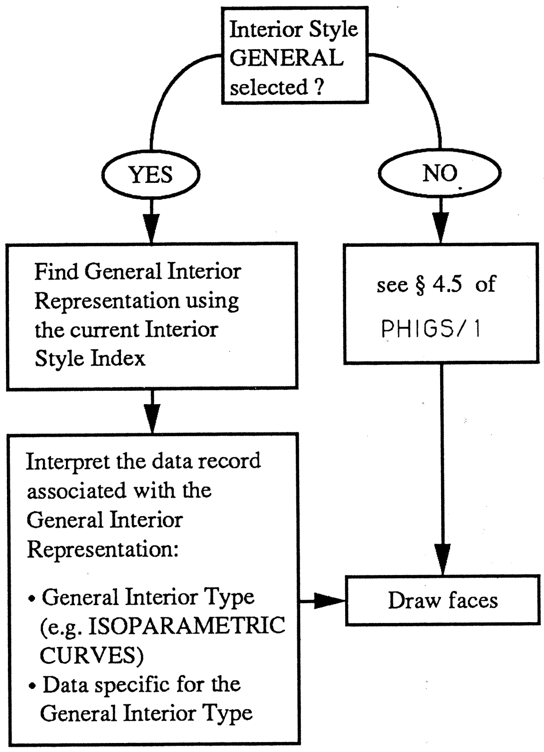


Figure 1. Mechanism of Interior Style GENERAL

be the parametric representation of the surface as defined in section 2. Then $\vec{S}(u,v)$ is visualised in the following way (see figure 3a):

1) The tessellated surface is drawn in one of the Interior Styles HOLLOW, SOLID or EMPTY;

2) The parametric curves

$$\vec{S}(u_i,v),\ i=1,\cdots,M; v_{min} \leqslant v \leqslant v_{max};$$

and

$$\vec{S}(u,v_j),\ j=1,\cdots,N; u_{min} \leqslant v \leqslant u_{max};$$

are drawn using the polyline attributes and the Curve Approximation Method; these are the so-called *isoparametric curves;* u_i and v_j can be selected either *uniformly between the parameter bounds* ($u_i = u_{min} + i/M_u(u_{max} - u_{min})$, $i = 0, \cdots, M_u$, v_j defined similarly) or *uniformly between the knots of the (u,v)-grid* ($u_{ik} = u_i + k/M_u(u_{i+1} - u_i)$, $k = 0, \cdots, M_u$, where u_i are the knots for the u-grid; v_{jk} are defined similarly);

The Interior Style used, M_u, N_v and the selection criterion of the points (uniform between the parameter bounds or between the knots) are contained in the data record of the General Interior Type.

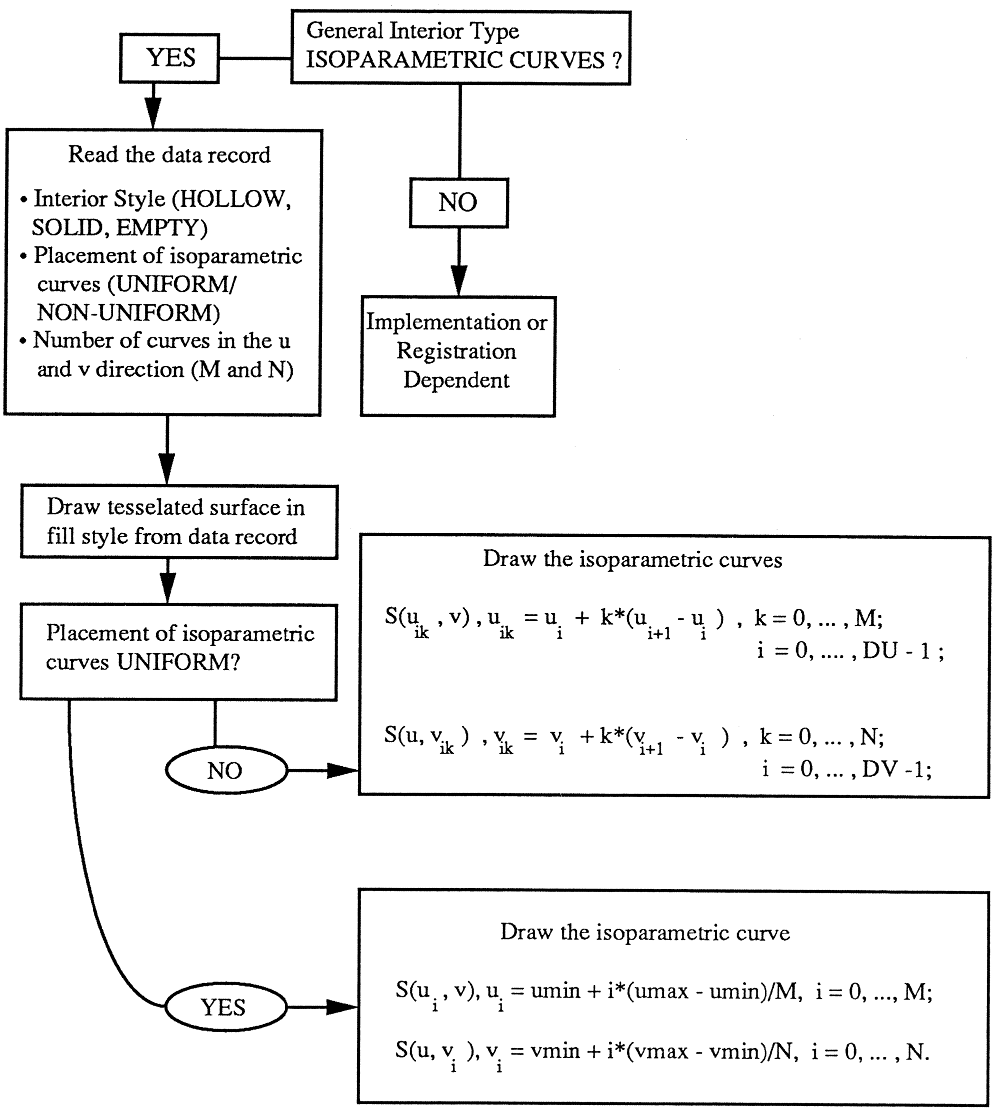


Figure 2. Mechanism of General Interior Type ISOPARAMETRIC CURVES

5. Possible constructions of patterns and hatches in PHIGS PLUS by means of the Interior Style GENERAL

From a geometrical point of view, the wire-frame model described in the previous section could be interpreted as a mapping of the 2D lines

$$u = u_i,\ i=1, \cdots, M_u;$$

$$v = v_j,\ j=1, \cdots, N_v;$$

in the (u-v) modelling space to parametric curves in 3D or 4D modelling space. In fact, a grid of

horizontal and vertical lines is mapped to a sequence of lines which determine the shape of the parametric surface (see figure 3a).
From this example, however, we can see that the mechanism of the Interior Style GENERAL offers the application programmer a unique tool to draw curved surfaces in a more picturesque style than PHIGS now admits. The mechanism described makes it possible to map arbitrary 2D figures to 3D surfaces.
Let us sketch some examples of General Interior Types, which can be implemented:

1) PATTERN MAPPING; (see figure 3b);
   a pattern is defined in the (u-v) parameter space and this pattern is mapped to the parametric surface;
   The basic data for this patterning could be defined by the following data record:
   a) a pattern rectangle in the (u-v) parameter space; this rectangle could be given by two points or by a reference point and the pattern size;
      REMARK: another possibility to define the pattern rectangle is the use of *weight vectors* to denote the relative position of the corners of the pattern box; thus, the weight vector (ω_1, ω_2) denotes the point (u,v) by

$$u = (1-\omega_1)u_{min} + \omega_1 u_{max},$$

$$v = (1-\omega_2)v_{min} + \omega_2 v_{max};$$

      in the domain $[u_{min}, u_{max}; v_{min}, v_{max}]$;
   b) a pattern representation given directly or by a pattern index;

2) HATCH MAPPING; (see figure 3c);
   a hatch is defined in the (u-v) space and the hatch lines are mapped to the parametric surface; the data record could consist of the following items;
   a) one of the Interior Styles HOLLOW, SOLID or HATCH;
   b) the hatch style;
   c) the number of horizontal, vertical or (cross-) diagonal hatches in the (u-v) parameter space;
   REMARK: Note that ISOPARAMETRIC CURVES is a special case of HATCH MAPPING.

6. Some more possible General Interior Types

The patterning and hatching styles described above are only two more examples of how the Interior Style GENERAL could be exploited in PHIGS PLUS. In fact, the Interior Style GENERAL is a kind of Pandora's box, because it allows virtually every 2D picture to be mapped onto a 3D surface.
In this section, we give some more nice examples. Not all of them may be suitable for standardisation in PHIGS PLUS, but they show how PHIGS PLUS could be used not only for curved surfaces but for drawings on curved surfaces as well.

3) TEXT MAPPING;
   a text is defined in the (u-v) rectangle and this text is mapped to the parametric surface; the data record could consist of the following items;
   a) the text and the text position;
   b) some text attributes;
   in implementing this General Interior Type, one could for example use the B-spline representation of the ASCII characters [6];
   REMARK: As in the case of PATTERN MAPPING, the geometric information could be given independent of the metric of the (u-v)-space, for example the text position could be given by two weight factors between 0 and 1;
4) PARAMETRIC CURVE;
   a parametric curve is defined in the (u-v) parameter space and this curve is mapped to the parametric surface; the data record could consist of the following items;
   a) the definition of the curve; the curve could be defined by the class (NURB, Piecewise Bézier,

inter alia), a grid, and the control points in (u-v) space;

b) some polyline attributes;

5) VERTEX MARKING;

the surface is drawn in one of the Interior Styles SOLID, HOLLOW or EMPTY and the vertex points $\vec{S}(u_i, v_j)$ are marked; for the marking one can either use the current POLYMARKER attributes or one can define them in the General Interior Type data record;

6) TRIMMING LOOP;

a sequence of curves in the (u-v) parameters space defines a curved clipping boundary of the surface;

REMARK: This TRIMMING LOOP is already part of the definition of the NON-UNIFORM B-SPLINE in PHIGS PLUS, but it could also be defined this way.

The above examples, sketchy as they may be, show the numerous possibilities that are offered by the new Interior Style GENERAL.

7. The Surface Approximation Method Criterion and the General Interior Type

The two aspects Surface Approximation Method Criterion and Interior Style are two orthogonal attributes of a Non-Uniform B-Spline Surface. They could, however, interfere, if the Interior Style is GENERAL. In this section, we give some examples, which illustrate this point. Also, we outline how this drawback can be repaired.

1) The General Interior Type is PATTERN MAPPING and the Surface Approximation Method Criterion is CONSTANT PARAMETRIC BETWEEN KNOTS;

Either aspect of the parametric surface involves a partition of the domain $[u_{min}, u_{max}; v_{min}, v_{max}]$:

a) Because of the Surface Approximation Method Criterion, the surface is subdivided into M*N surface segments, each of which is approximated by a set of planar faces (for example two triangles or one quadrilateral).

b) On the other hand, because the Interior Style is GENERAL and the General Interior Type is PATTERN MAPPING, the surface is subdivided into M*N surface segments, each of which is rendered with a constant General Colour.

Let us denote these two subdivisions by $\Delta_T(u)*\Delta_T(v)$ and $\Delta_P(u)*\Delta_P(v)$, respectively.

Since in general $\Delta_T(u)*\Delta_T(v)$ will be no subpartition of $\Delta_P(u)*\Delta_P(v)$ (the possibility that $\Delta_P(u)*\Delta_P(v)$ is a subpartition of $\Delta_T(u)*\Delta_T(v)$ is not considered here, because $\Delta_T(u)*\Delta_T(v)$ will usually be fine and $\Delta_P(u)*\Delta_P(v)$ will usually be coarse), an efficient rendering of the surface could be hampered: several of the subrectangles of $\Delta_T(u)*\Delta_T(v)$ would be mapped to sets of faces with different colours on them.

One solution could be the addition of the grid-points of $\Delta_P(u)*\Delta_P(v)$ to those of $\Delta_T(u)*\Delta_T(v)$ (in which case each subrectangle of the extended $\Delta_T(u)*\Delta_T(v)$ is mapped onto a set of faces with the same colour), but in that case the Surface Approximation Method Criterion is no longer CONSTANT PARAMETRIC BETWEEN KNOTS, which could result in a slower rendering of the surface, because the nice properties of this Surface Approximation Method Criterion are lost.

Another and possibly more efficient solution could be the introduction of a new Surface Approximation Method Criterion: CONSTANT PARAMETRIC BETWEEN PATTERN KNOTS. This Surface Approximation Method Criterion means that each rectangle of $\Delta_P(u)*\Delta_P(v)$ is uniformly partitioned into M*N subrectangles, each of which is mapped to an approximation of the corresponding surface segment.

An advantage of this method is that the surface can be rendered rather efficiently:

for each pattern cell, the corresponding surface segment is approximated by a rectangular array of (a set of) planar faces of the same colour; these processes could easily be done in parallel;

This process can easily be done in parallel.

A draw-back of this method is that the current Surface Approximation Method Criterion is silently overruled and the orthogonality of the aspects Interior Style and Surface Approximation Method Criterion is violated. A remedy to this violation could be the integration of this special Surface Approximation Method Criterion into the data record of the General Interior Type PATTERN MAPPING, possibly with a flag denoting whether the current Surface Approximation Method Criterion is overruled or not.

2) The General Interior Type is ISOPARAMETRIC CURVES with Interior Style SOLID and the Surface Approximation Method Criterion is CONSTANT PARAMETRIC BETWEEN KNOTS; In general , the curves of the wire-frame do not lie exactly on the tessellated surface. A solution could be the use of a Surface Approximation Method Criterion based on the partition $\Delta_I(u)*\Delta_I(v)$ of $[u_{min}, u_{max}; v_{min}, v_{max}]$, for example a further partition of $\Delta_I(u)*\Delta_I(v)$.
This solution would have the advantage that the tessellation of the surface segments and the drawing of the isoparametric curves can be done segment by segment: each rectangle of $\Delta_I(u)*\Delta_I(v)$ is mapped to the corresponding surface segment by creating a rectangular array of approximating faces. Also in this case, these processes can be done in parallel.

8. User Example

To illustrate the possibilities of the Interior Style GENERAL, we give the simple quadric example

$$X(u,v) = (1 - u^2)(1 - v^2);$$

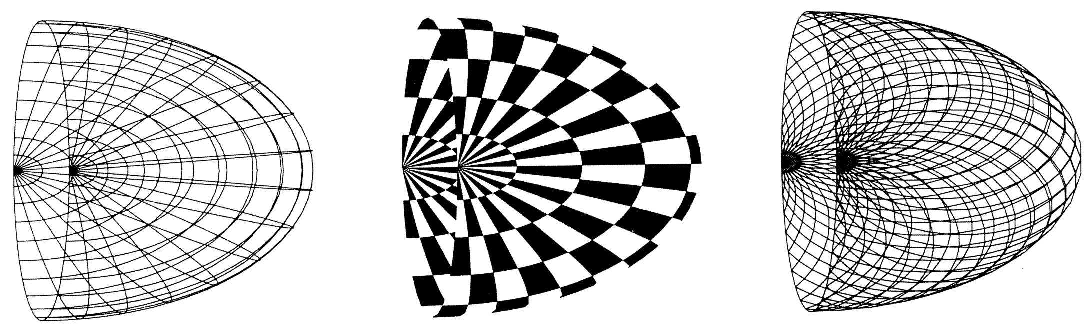
$$Y(u,v) = 2(1 - u^2)v;$$

$$Z(u,v) = 2u(1 + v^2);$$

$$W(u,v) = (1 + u^2)(1 + v^2);$$

$$-1 \leqslant u \leqslant +1 \; ; \; -1 \leqslant v \leqslant +1;$$

which defines a half sphere in 4D homogeneous coordinates. This biquadratic function can easily be represented by a (piecewise) quadratic Bèzier polynomial.



(a) (b) (c)

Figure 3. Plot of half sphere with the General Interior Types ISOPARAMETRIC CURVES (3a),PATTERN MAPPING (3b), and HATCH MAPPING (3c)

In figure 3, the sphere is drawn with the General Interior Types ISOPARAMETRIC CURVES, PATTERN MAPPING, and HATCH MAPPING. The figures were generated by a C application program on top of an implementation of GKS-3D/C [8] using a PostScript ® driver with a high-quality laser printer as output device.

9. Recent developments in PHIGS PLUS

Since August 1989, some changes in PHIGS PLUS have been made:

1) The Interior Style GENERAL has been replaced by a new Surface Attribute: Surface Characteristics;
2) The NURB curves and surfaces are now the only curved primitives supported by PHIGS PLUS;
3) The fourth components of the control points in 4D homogeneous coordinates (the C_{4k} and S_{4kl} from section 2) have only positive values;
4) The primitive NURB SURFACE WITH DATA has been added to PHIGS PLUS; this new primitive supports inter alia the mapping of a non-uniform colour grid onto a curved surface.

These changes, however, only affect the organization of PHIGS PLUS, not the functionality, as far as it concerns curved surfaces and lines.

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